

# Introduction to Computer Vision for Robotics

**AE640A** Autonomous Navigation

12<sup>th</sup> March, 2019



# Lecture Outline

- Features
  - Motivation for feature points
  - Harris Detector
- SIFT



# Features



# Features



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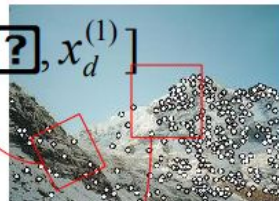


# Features

- 1) Detection: Identify the interest points



$$\mathbf{x}_1 = [x_1^{(1)}, \boxed{?}, x_d^{(1)}]$$



- 2) Description: Extract vector feature descriptor surrounding each interest point.



$$\mathbf{x}_2 = [x_1^{(2)}, \boxed{?}, x_d^{(2)}]$$

- 3) Matching: Determine correspondence between descriptors in two views

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# Features

- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

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# Features

- We want to detect (at least some of) the same points in both images.



**No chance to find true matches!**

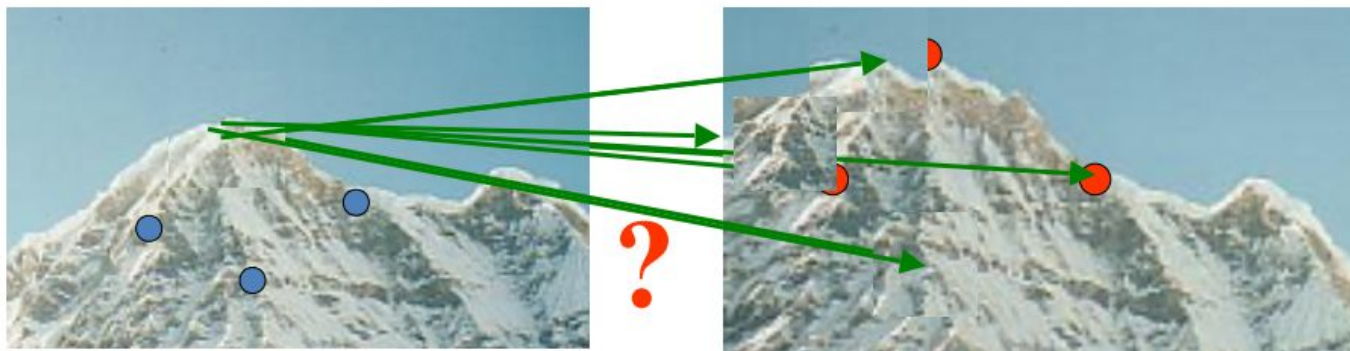
- Yet we have to be able to run the detection procedure *independently* per image.

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# Features

- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.

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# Features

**Look for image regions that are unusual**

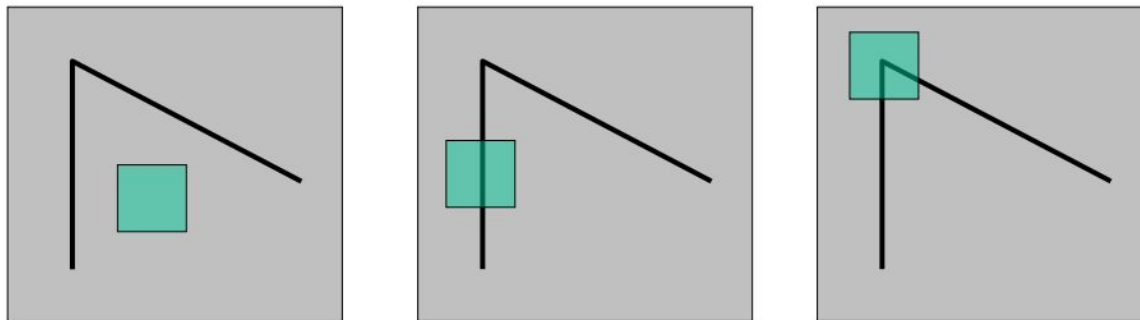
- 1) Lead to unambiguous matches in other images

**How to define “unusual”?**

# Features

Suppose we only consider a small window of pixels

1) What defines whether a feature is a good or bad candidate?



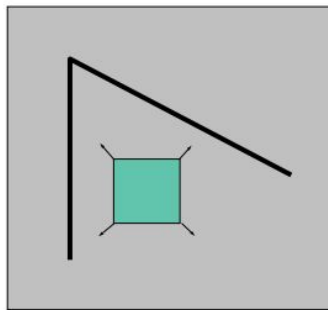
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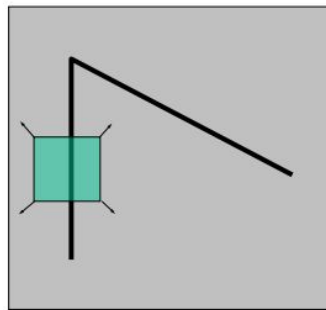
# Features

## Local measure of feature uniqueness

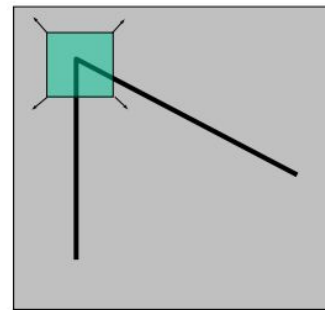
- 1) How does the window change when you shift it?
- 2) Shifting the window in *any direction* causes a *big change*



**“flat” region:**  
no change in all  
directions



**“edge”:**  
no change along  
the edge direction



**“corner”:**  
significant change  
in all directions

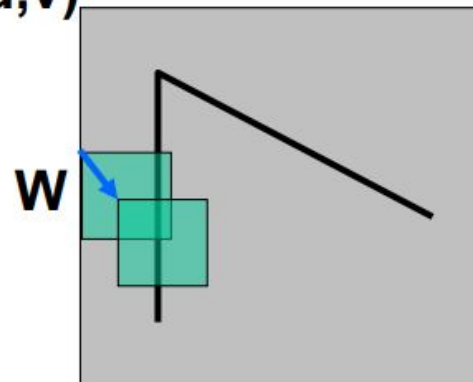
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# Features

**Consider shifting the window  $W$  by  $(u,v)$**

- 1) how do the pixels in  $W$  change?
- 2) compare each pixel before and after by summing up the squared differences (SSD)
- 3) this defines an SSD “error” of  $E(u,v)$ :



$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

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# Features

## Taylor Series expansion of I:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

**If the motion (u,v) is small, then first order approx is good**

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

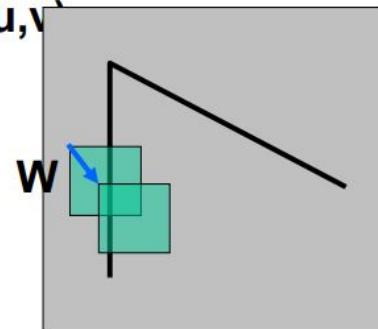
$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$

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# Features Consider shifting the window $W$ by $(u, v)$

- 1) how do the pixels in  $W$  change?
- 2) compare each pixel before and after by summing up the squared differences
- 3) this defines an “error” of  $E(u, v)$ :



$$\begin{aligned} E(u, v) &= \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2 \\ &\approx \sum_{(x, y) \in W} [I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x, y)]^2 \\ &\approx \sum_{(x, y) \in W} \left[ [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2 \end{aligned}$$

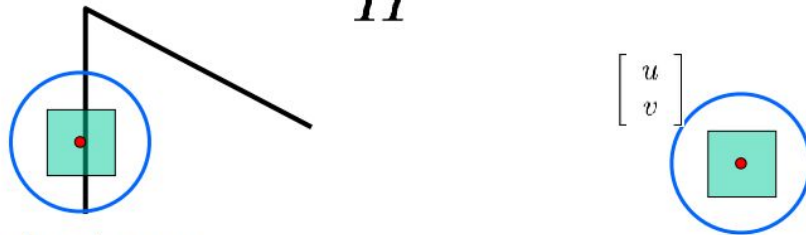
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# Features

This can be rewritten:

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$



**For the example above**

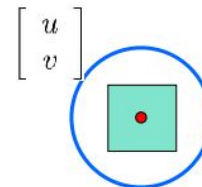
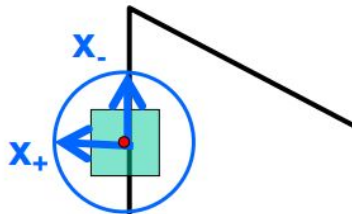
- 1) You can move the center of the green window to anywhere on the blue unit circle
- 2) Which directions will result in the largest and smallest E values?
- 3) We can find these directions by looking at the eigenvectors of  $H$

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# Features

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$



## Eigenvalues and eigenvectors of H

- 1) Define shifts with the smallest and largest change (E value)
- 2)  $x_+$  = direction of largest increase in E.
- 3)  $\lambda_+$  = amount of increase in direction  $x_+$
- 4)  $x_-$  = direction of smallest increase in E.
- 5)  $\lambda_-$  = amount of increase in direction  $x_+$

$$Hx_+ = \lambda_+ x_+$$

$$Hx_- = \lambda_- x_-$$

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# Features

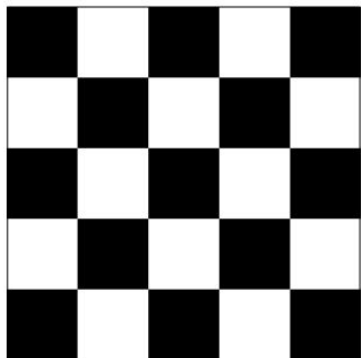
How are  $\lambda_+$ ,  $x_+$ ,  $\lambda_-$ , and  $x_-$  relevant for feature detection?

1) What's our feature scoring function?

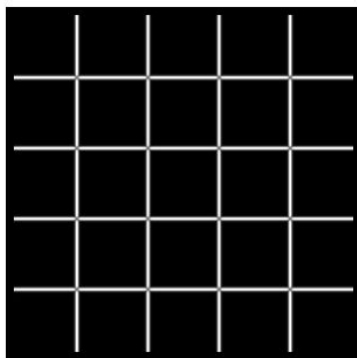
Want  $E(u,v)$  to be *large* for small shifts in *all* directions

1) the *minimum* of  $E(u,v)$  should be large, over all unit vectors  $[u \ v]$

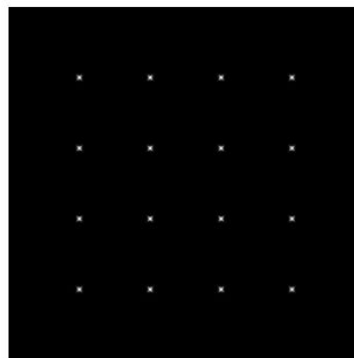
2) this minimum is given by the smaller eigenvalue ( $\lambda_-$ ) of  $H$



$I$



$\lambda_+$



$\lambda_-$

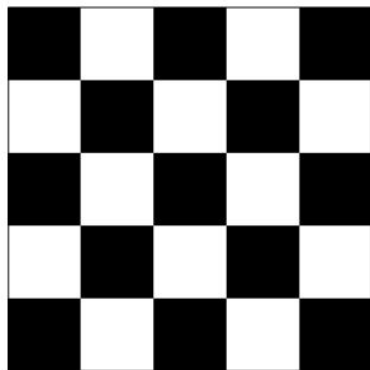
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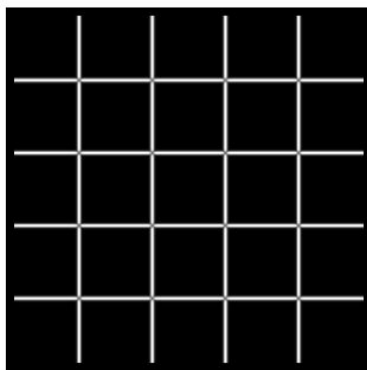
# Features

## Here's what you do

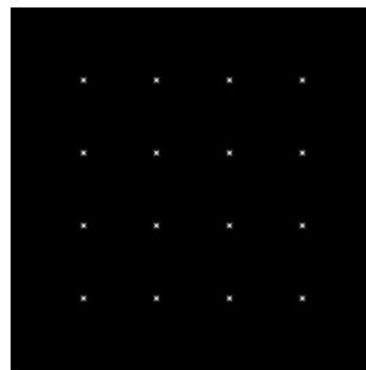
- 1) Compute the gradient at each point in the image
- 2) Create the  $H$  matrix from the entries in the gradient
- 3) Compute the eigenvalues.
- 4) Find points with large response ( $\lambda_+ > \text{threshold}$ )
- 5) Choose those points where  $\lambda_+$  is a local maximum as features



$I$



$\lambda_+$



$\lambda_-$

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# Features

$\lambda_+$  is a variant of the “Harris operator” for feature detection

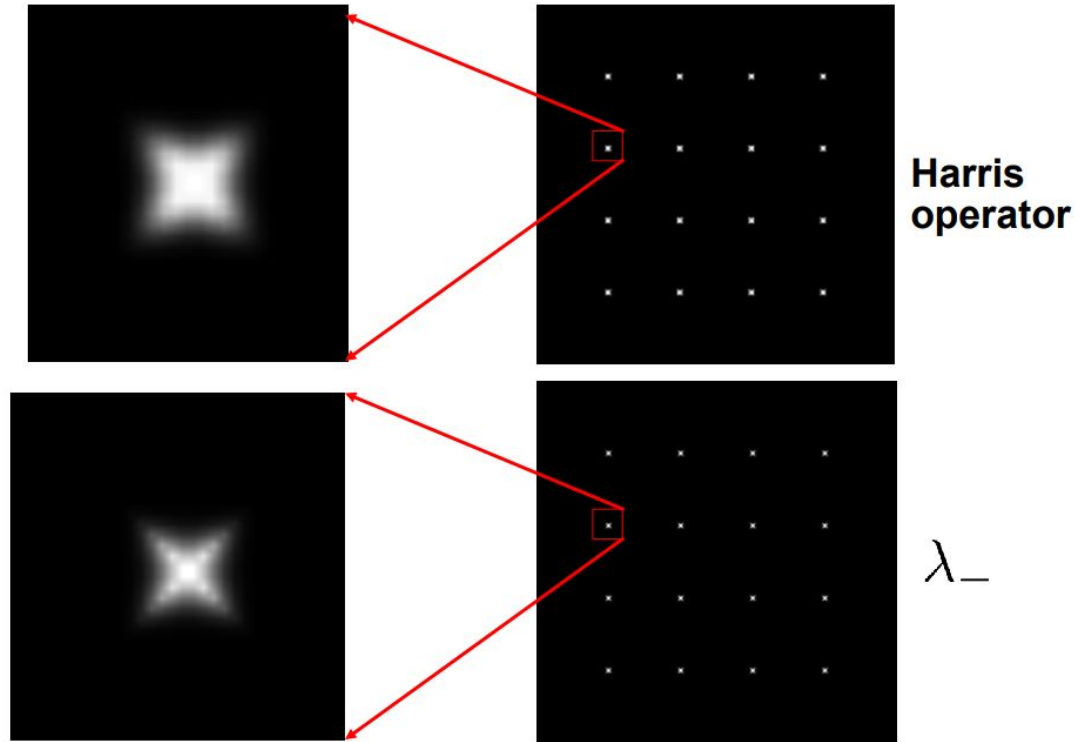
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$
$$= \frac{\text{determinant}(H)}{\text{trace}(H)}$$

- 1) The *trace* is the sum of the diagonals, i.e.,  $\text{trace}(H) = h_{11} + h_{22}$
- 2) Very similar to  $\lambda_+$  but less expensive (no square root)
- 3) Called the “Harris Corner Detector” or “Harris Operator”
- 4) Lots of other detectors, this is one of the most popular

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# Features



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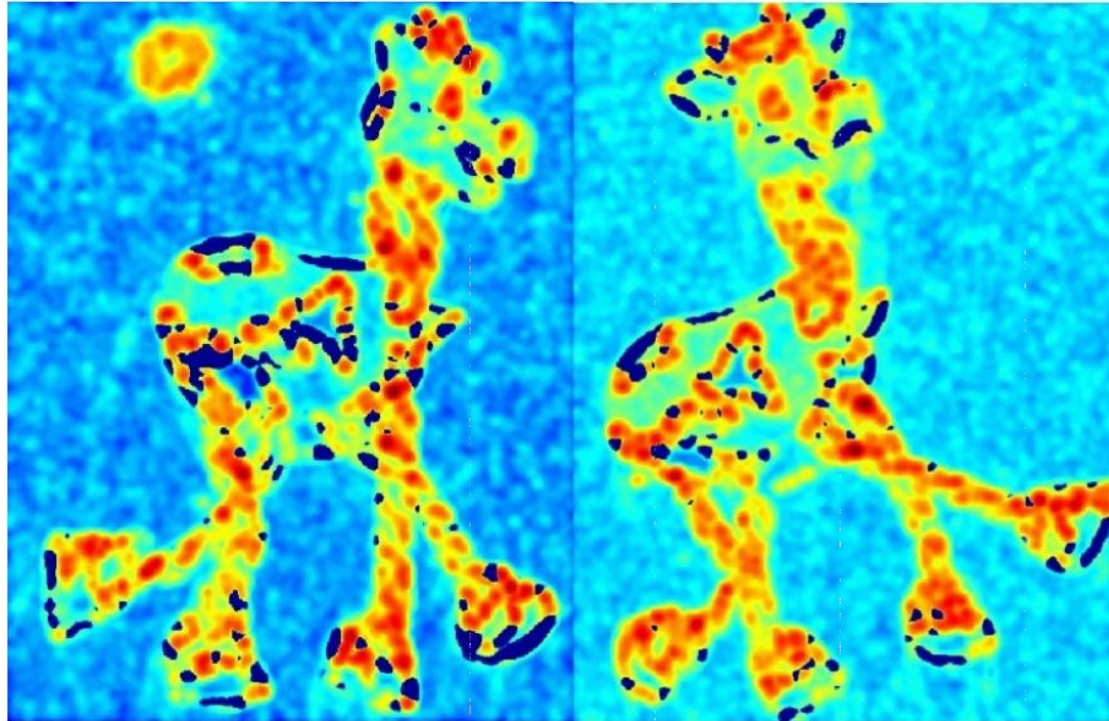
# Features



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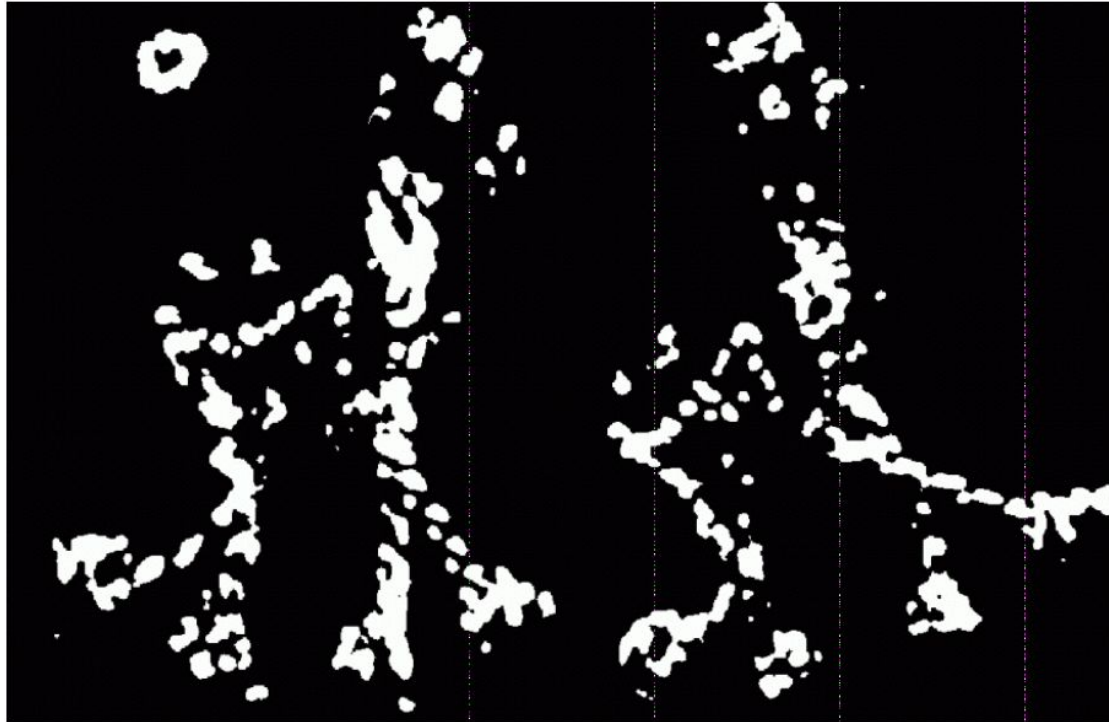
# Features



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# Features



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# Features



Credits: Vinay Nambodiri





# Features



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# SIFT

